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Waves of Optimism and Pessimism

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Waves of Optimism and Pessimism

Beatrice Pataracchia*

Abstract

We consider a simple consumption-based asset pricing model with two types of investors who have access to the same observations but who use different updating rules to infer information about the growth state of the economy. In particular, we consider an optimistic and pessimistic group of agents who use distorted Bayesian updating rules. The aim of the work is to understand to what extent the interaction of such distorted Bayesian rules can explain low and medium frequency characterization of dynamics movements observed in the price dividend ratios and can give rise endogenously to waves of pessimism and optimism which are associated with sustained asset price booms and busts. The analysis shows that heterogeneity in ambiguity loving/aversion preferences appears to be an important factor to capture medium-frequency waves observed on asset prices.

Keywords: Asset Pricing, Heterogeneous Agents, Optimism, Pessimism

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1 Introduction

Standard consumption-based asset pricing models have difficulty in explaining many characteristics of financial markets. The literature has documented typical puzzles implied by standard representative agent model with time separable utility: the observed higher returns of risky assets compared to risk-free investment, which could be rationalized only with an implausible high degree of relative risk aversion (Mehra and Prescott, 1985), the unrealistic implication of high risk free returns (Weil, 1989) and the excess volatility observed in stock prices, which is difficult to replicate in models (Shiller, 1981). Such failures have motivated an important effort in the financial literature to provide plausible explanations. One direction has focused on the representative investor who fears uncertainty about the model used and/or tries to learn to resolve such uncertainty. The idea of knightian uncertainty, precautionary behaviors and the need of robustness against model misspecification, have been proposed as a possible explanation for high returns of risky assets (Hansen and Sargent, 2005; Maenhout, 2004; Anderson, Hansen and Sargent, 2003; Leippold, Trojani and Vanini, 2008). Another stream has explored the implication of the interaction of heterogeneous agents and beliefs and focused on the conditions for long run survival of the agents with the most correct beliefs (Blume and Easley, 1992; Sandroni, 2000). It is known that the interaction of agents with different beliefs is able to capture some chaotic pricing behavior observed in data, such as clustering volatility, bubbles and fat tails. From Brock and Hommes (1997, 1998) an increasing stream of literature has focused on a computationally

intensive approach based on the "artificial stock market", which consists of simulating many interacting agents who can exchange assets repeatedly in time. One drawback of this literature is the high parameterization on which it relies. This naturally casts problems of estimation, validation and robustness (Amilon, 2008, Li, Donkers and Melenberg 2010).

In order to overcome this problem, we consider a very simple model in line with Sandroni (2000) who proposes a model where agents have different beliefs about consumption growth's realizations. He investigates the conditions for the convergence to rational expectations stating that if agents have the same intertemporal discount factor and the same utility function, those who make wrong predictions are driven out of the market. Our goal is different. We do not investigate long-run rational expectation conditions, rather we want to study the implications of a long-run persistence of heterogeneity of beliefs where all the agents have access to the same information. Such simple framework allows us to study the effect of heterogeneity of beliefs due to the disagreement in the interpretation of new information.

Disagreement caused by psychological attitudes has been disregarded by traditional economics who fails to take into account the extent to which people are also guided by noneconomic motivations. Akerlof and Shiller (2009) reasserts the necessity to consider the role of "animal spirits" in economic analysis. *"The idea that economic crises [...] are mainly caused by changing thought patterns goes against standard economic thinking. But the current crisis bears witness to the role of such changes in thinking. It was caused precisely by our changing confidence [...] and especially by changing stories of the nature of the economy"* (Akerlof and Shiller, 2009, p.4). The

uncertainty about the unstable economic conditions after the severe 2008 financial crisis has been an example of the relevance of this aspect: investors were not wondering about the access of information or measurement issues, rather they have tried to understand the underlying characteristics of the economy, discussing about whether and when the recession period would be over. Temporary positive observations do not necessarily resolve the uncertainty: some analysts have interpreted them as a clear sign of recovery, others were more cautious. In this work, we want to analyze such disagreement in the interpretation of information and we study its effects on asset prices. The heterogeneity in our context is characterized by the attitude toward optimistic and pessimistic interpretation of the information.

Changing confidence in our model is represented by time-varying fraction of optimists and pessimists. We impose distortions in a way such that all the agents are *equally wrong* about future consumption growth realizations. In this way we preserve heterogeneity in the long run and focus on the implications of the dynamics of prices. The consumption shares, and their effects on prices, inversely depend on how wrong investors' forecasts have been compared to consumption realizations. With this simple framework, we want to model the idea that investors do not simply have difficulties in accessing information, rather they can interpret it differently, depending on their personal attitudes. In particular, we ask ourselves whether this simple framework is able to capture the main observed low-medium frequency movements in asset prices. Indeed, contrary to the hypothesis of informationally efficient markets (Fama 1970, 1991), which implies stock returns close to being unpredictable and prices close to a random walk, a central

fact driving forecastable long-horizon returns is that the price-dividend ratio is far from being a random walk: it has persistent fluctuations and is excessively volatile. A part from the macroeconomic idea that level and movement of risk premia are important for understanding the business cycle, the attention to price-dividend ratio is also motivated by the empirical evidence that the price-dividend ratios appear to be able to predict substantial amounts of stock return variation.

We show that introducing a very simple form of distortion of beliefs in the way described above, can have a notable effect on the implied price dynamics: heterogeneity of beliefs amplifies prices and fluctuations so to capture and reproduce the medium-frequency waves of prices observed in the data.

In order to analyze the sole effect of heterogeneity and to make the interpretation of the results easy, we assume that the consumption growth is modeled as a simple discrete regime switching process. Even in such an unrealistic case, the implied price-dividend ratio is very volatile compared to the distortion-neutral case and the dynamics of the implied fluctuations are able to match, qualitatively, the observed fluctuations.

Finally, we show that testable Euler moment conditions can be defined introducing only one additional degree of freedom and model parameters can be estimated using a Generalized Method of Moments technique. We find that estimates of the distortion are typically positive, but present very high standard errors. Such failure, however, cannot be attributed solely to our model as it is a common feature of simple consumption based asset pricing models (Campbell and Cochrane, 2000).

We conclude that the ingredient of heterogeneity among economic agents remains fundamental to understand and forecast price fluctuations. The additional degree of freedom is due to the fact that we allow agents to have a personal and heterogeneous attitude in interpreting information and exchanging assets. The agents neither learn nor do they imitate others' behavior: they are just different in the way in which they interpret reality even if they can share the same degree of risk aversion or the same discount factor.

The remainder of this paper is structured as follows. Section 2 describes the model and the agents' optimization problem. Section 3 presents a quantitative analysis based on the model estimates, focusing, especially on the historical implied dynamics of the price-dividend ratio for different distortion degrees. In Section 4, the testable model restrictions are derived and Section 5 concludes.

2 The model

We study a simple dynamically complete market with two long-lived groups of agents, two assets, two states of nature and one single consumption good. Let T be the set of natural numbers. At period $t \in T$, the agents observe the state of nature of the consumption growth process, $y_t = \log \frac{C_t}{C_{t-1}}$. The set of states of nature is given by $Y \equiv \{y_h, y_l\}$, with $y_h > y_l$. The true stochastic process of the states of nature is given by a Markovian structure so that

$$\log \frac{C_{t+1}}{C_t} = y_{t+1} = y(S_{t+1}), \quad (1)$$

where S_{t+1} is a two-state Markov chain with associated transition matrix $P = \{p_{ij}\}_{2 \times 2}$ where $p_{ij} = \text{prob}[S_{t+1} = j | S_t = i]$, with $i, j = h, l$. In the remainder of the paper, we denote $y(h)$ with y_h and $y(l)$ with y_l .

The investors can choose among two assets: asset 1 pays the aggregate dividend D_t if y_h realizes, and zero otherwise. Asset 2 pays D_t if y_l realizes and zero dividends if y_h realizes. As standard, we assume that aggregate dividends correspond to aggregate consumption, $D_t = C_t$. The assumption of two states can provide a simple, yet realistic, description of how economic agents process information in reality: agents are, by nature, are mostly influenced by the qualitative aspects (good versus bad) of new information more than by the precise quantitative data.

We consider two groups of agents who have different psychological attitudes: they can be optimistic or pessimistic. They have access to the same information y_t , but they bias their beliefs so that the optimistic agents tend to believe that good periods are more likely and the pessimistic ones bias their beliefs so to increase the probability of the economy being in a low-mean state.

Let T be the set of natural numbers. At time $t \in T$, the history of realizations of the consumption growth, $y^t = \{y_0, y_1, \dots, y_t\}$, is a commonly available information: agents observed the state and update their beliefs about the next observation. The subjective beliefs of agent i are defined by the vector $\pi_t^i = \begin{bmatrix} \pi_t^i(h) & \pi_t^i(l) \end{bmatrix}$, where $\pi_t^i(l) = 1 - \pi_t^i(h) = \text{prob}^i(S_{t+1} = l | y^t)$ is the prior probability of state l in period $t + 1$ according to agent i after she observes y_t . The full-bayesian prior belief of state s in time $t + 1$ is defined as

$$\pi_t(s) = \sum_{j \in \{h,l\}} p_{js} 1_{\{j\}}.$$

Brandt et al. (2004) provide a simple way to model subjective attitudes: the optimistic belief of the high-mean state, π_t^o , is defined as:

$$\pi_t^o(h) = (1 - \omega^o) \pi_t(h) + \omega^o$$

where $\omega^o \in (0, 1)$ is a parameter which measures the degree of the optimistic distortion. Similarly, the pessimistic belief is described as

$$\pi_t^p(h) = (1 - \omega^p) \pi_t(h)$$

where $\omega^p \in (0, 1)$. The definitions of $\pi_t^o(l)$ and $\pi_t^p(h)$ follow immediately. It is also easy to check that $\pi_t^o(h)$ can be obtained by distorting the belief about the transition probabilities:

$$\pi_t^o(s) = \sum_{j \in \{h,l\}} p_{js}^o 1_{\{j\}},$$

where

$$p_{hh}^o = (1 - \omega^o) p_{hh} + \omega^o$$

and

$$p_{lh}^o = (1 - \omega^o) (1 - p_{ll}) + \omega^o.$$

We can, therefore, think of the optimistic (pessimistic) agents also as those

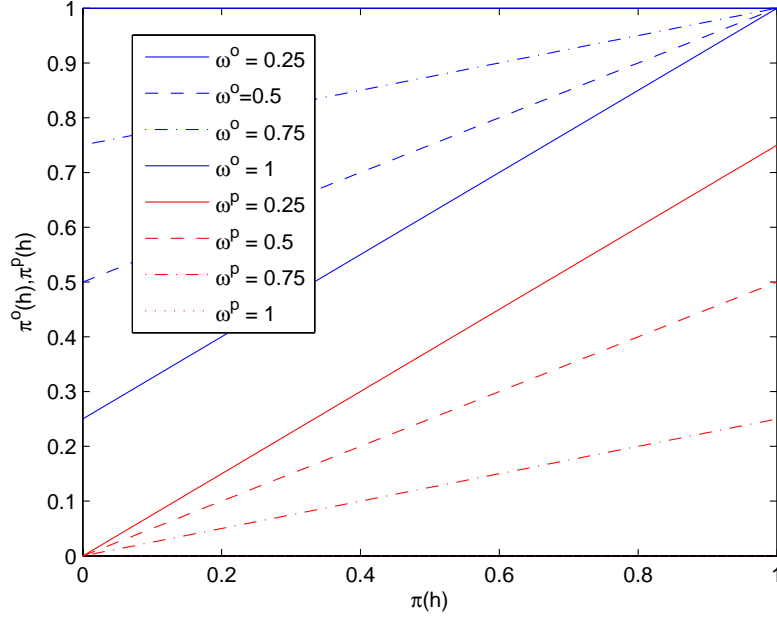


Figure 1: Optimistic (blue lines) and pessimistic (red lines) distortions of beliefs.

who consider the good (bad) states more persistent. Figure 1 shows the effect of the distortion of the belief of the high growth state, plotted on the x -axis. The optimistic (pessimistic) beliefs are always above (below) the 45-degree line and the greater are the distortion parameters, ω^o and ω^p , the farther are the subjective beliefs from the unbiased ones.

Agents' optimization problem We can find the equilibrium allocation by posing a Pareto problem for a fictitious social planner, who attaches nonnegative Pareto weights $\lambda_i, i = o, p$ on the consumers and maximizes the social utility function W :

$$W = \lambda_o U_o + \lambda_p U_p \quad (2)$$

where U_i is the agent i 's utility functional:

$$\begin{aligned} U_i &= \mathbb{E}_0^i \sum_{t=0}^{\infty} \beta^t u(C_t^i) \\ &= \sum_{t=0}^{\infty} \sum_{y^t} \beta^t \text{prob}^i(y^t|y_0) u(C_t^i(y^t)), \end{aligned} \quad (3)$$

where \mathbb{E}_0^i is the mathematical expectation operator conditioned on y_0 , and $\text{prob}^i(y^t|y_0)$ represents the agent i 's conditional probability of observing the realized history y^t prior to any observation. We assume that trading occurs after observing y_1 , and we set $\text{prob}^i(y^0|y_0) = 1$. The utility function $u(\cdot)$ is supposed to be identical among agents. Furthermore, $u(\cdot)$ is an increasing concave function of consumption $C \geq 0$ and satisfies the Inada conditions.

The maximization is subject to the time t , history y^t budget constraint:

$$\begin{aligned} \sum_{i \in \{o,p\}} C_t^i(y^t) &= \\ &= \sum_{i \in \{o,p\}} \left\{ \sum_{m=1}^2 \left[(P_t^m(y^t) + D_t^m(y^t)) x_{t-1}^{i,m}(y^{t-1}) - P_t^m(y^t) x_t^{i,m}(y^t) \right] \right\}, \end{aligned} \quad (4)$$

where $x_t^{i,m}(y^t)$ denotes the quantity of asset m chosen by the investor i at time t , $P_t^m(y^t)$ and $D_t^m(y^t)$ are the time t price and the paid dividend of asset m , respectively. Markets clear when

$$\begin{aligned}
C_t^o(y^t) + C_t^p(y^t) &= C_t(y^t) = D_t(y^t), \forall t \in T, \\
x_t^{o,m}(y^t) + x_t^{p,m}(y^t) &= 1, \forall t \in T, m = 1, 2,
\end{aligned} \tag{5}$$

where $C_t^o(y^t)$ and $C_t^p(y^t)$ are the consumption allocations among the optimists and pessimists, respectively, while $C_t(y^t)$ and $D_t(y^t)$ denote the aggregate consumption and the aggregate dividend.

Let $\mu_t(y^t)$ be a nonnegative multiplier on the budget constraint. The Lagrangian of the optimization problem (2)-(3) can then be formed:

$$\mathcal{L} = \sum_{t=0}^{\infty} \sum_{y^t} \left[+\mu_t(y^t) \left(\sum_{i \in \{o,p\}} \left\{ \sum_{m=1}^M \left[P_t^m(y^t) x_{t-1}^{i,m}(y^{t-1}) + D_t^m(y^t) x_{t-1}^{i,m}(y^{t-1}) - P_t^m(y^t) x_t^{i,m}(y^t) \right] - C_t^i(y^t) \right\} \right) \right].$$

The first order conditions of maximizing \mathcal{L} with respect to $C_t^i(y^t)$ and $x_t^{i,m}(y^t)$ are

$$\lambda^i \beta^t \text{prob}^i(y^t|y_0) u'(C_t^i(y^t)) = \mu_t(y^t), \tag{6a}$$

$$\mu_t(y^t) P_t^m(y^t) = \sum_{y_{t+1}} \mu_{t+1}(y^{t+1}) (P_{t+1}^m(y^{t+1}) + D_{t+1}^m(y^{t+1})), \tag{6b}$$

$\forall m, \forall i, \forall t$. Substituting eq. (6a) into eq. (6b) we have:

$$P_t^m(y^t) = \beta \sum_{y_{t+1}} \text{prob}^i(y^{t+1}|y^t) \frac{u'(C_{t+1}^i(y^{t+1}))}{u'(C_t^i(y^t))} (P_{t+1}^m(y^{t+1}) + D_{t+1}^m(y^{t+1})). \tag{7}$$

In order to solve for prices we need to determine the equilibrium consumption shares or the ratio of the multipliers. Indeed, from eq. (6a), we derive the following condition on the ratio of multipliers:

$$\frac{prob^o(y^t|y_0) u'(C_t^o(y^t))}{prob^p(y^t|y_0) u'(C_t^p(y^t))} = \frac{\lambda^p}{\lambda^o}.$$

At time $t = 0$, the above condition implies that $\frac{\lambda^p}{\lambda^o} = \frac{u'(C_0^o(y_0))}{u'(C_0^p(y_0))}$. If we assume that the initial consumption shares of the two groups of agents are equal, we have:

$$prob^o(y_{t+1}|y_t) \frac{u'(C_{t+1}^o(y^{t+1}))}{u'(C_t^o(y^t))} = prob^p(y_{t+1}|y_t) \frac{u'(C_{t+1}^p(y^{t+1}))}{u'(C_t^p(y^t))},$$

or

$$\frac{u'(C_{t+1}^o(y^{t+1}))}{u'(C_{t+1}^p(y^{t+1}))} = \frac{prob^p(y_{t+1}|y_t) u'(C_t^o(y^t))}{prob^o(y_{t+1}|y_t) u'(C_t^p(y^t))}, \quad (8)$$

so that the optimal allocations are characterized by the property that, between any two commodities, all consumers share a common marginal rate of substitution. By backward substitution, we derive the fundamental equation for characterizing long-run equilibrium consumption:

$$\frac{u'(C_t^o(y^t))}{u'(C_t^p(y^t))} = \frac{u'(C_0^o(y_0))}{u'(C_0^p(y_0))} \frac{prob^p(y^t|y_0)}{prob^o(y^t|y_0)}. \quad (9)$$

When equal initial consumption shares are considered,¹ eq. (9) simplifies

¹Robustness studies have been conducted with different initial consumption endowment share. Results do not change significantly.

to:

$$\frac{u'(C_t^o(y^t))}{u'(C_t^p(y^t))} = \frac{prob^p(y^t|y_0)}{prob^o(y^t|y_0)}. \quad (10)$$

Then we can write:

$$\begin{aligned} u'(C_t^o(y^t)) &= \frac{prob^p(y^t|y_0)}{prob^o(y^t|y_0)} u'(C_t^p(y^t)) \\ &= \frac{prob^p(y^t|y_0)}{prob^o(y^t|y_0)} u'(C_t(y^t) - C_t^o(y^t)). \end{aligned}$$

In order to facilitate the comparison with similar studies and for its tractability, we consider the well known isoelastic CRRA utility functional:

$$u(C_t^o(y^t)) = \frac{(C_t^o(y^t))^{1-\alpha} - 1}{1-\alpha},$$

where α denotes the constant Arrow-Pratt measure of relative risk aversion.

The equilibrium condition on consumption shares then becomes:

$$C_t^o(y^t) = \left[\frac{prob^p(y^t|y_0)}{prob^o(y^t|y_0)} \right]^{-\frac{1}{\alpha}} [C_t(y^t) - C_t^o(y^t)]. \quad (11)$$

Denoting the ratio of beliefs of the representative agents of each group at time t , $\frac{prob^p(y^t|y_0)}{prob^o(y^t|y_0)}$, with $\eta_t(y^t)$, we can rewrite eq. (11) as follows:

$$\begin{aligned} C_t^o(y^t) &= \eta_t^{-\frac{1}{\alpha}}(y^t) [C_t(y^t) - C_t^o(y^t)] \\ &= \frac{1}{\eta_t^{\frac{1}{\alpha}}(y^t) + 1} C_t(y^t) \end{aligned} \quad (12)$$

and

$$C_t^p(y^t) = C_t(y^t) - C_t^o(y^t) = \frac{\eta_t^{\frac{1}{\alpha}}(y^t)}{\eta_t^{\frac{1}{\alpha}}(y^t) + 1} C_t(y^t), \quad (13)$$

which define the equilibrium shares of consumptions. As a direct consequence of Pareto optimality, the allocation of resources is optimal: each type of agents consumes more in the state which is considered more likely. The ratio of beliefs, η_t , indeed, summarizes the relative performance of the two representative agents: it is the ratio of the product of the probabilities that the two groups have attributed to the real observations. Furthermore, we observe that the consumption share depends exclusively on the ratio of beliefs and on the risk aversion parameter. Figure 2 plots the consumption share of the optimistic agent, $x_t^o = \frac{C_t^o(y^t)}{C_t(y^t)}$, as a function of the ratio of beliefs for different values of risk aversion. Higher risk aversion parameters smooth the consumption share as a function of the ratio of beliefs. When traders are nearly risk neutral, they take more extreme asset positions, so those with incorrect beliefs will be driven out of the market soon. In the extreme case of risk neutrality, the optimistic agent would consume everything for values of η_t less than one, while he would get nothing for values of η_t greater than unity, when all the aggregate consumption would be allocated among the pessimistic ones.

The equilibrium conditions (7), (12), and (13) allow us to determine the equilibrium prices and allocations. From the perspective of the optimistic agents, the equilibrium conditions for the price-dividend ratio of the asset m , $\frac{P_t^m}{D_t}$, is given by the following fixed-point recursive equations, where $\varphi_t^m =$

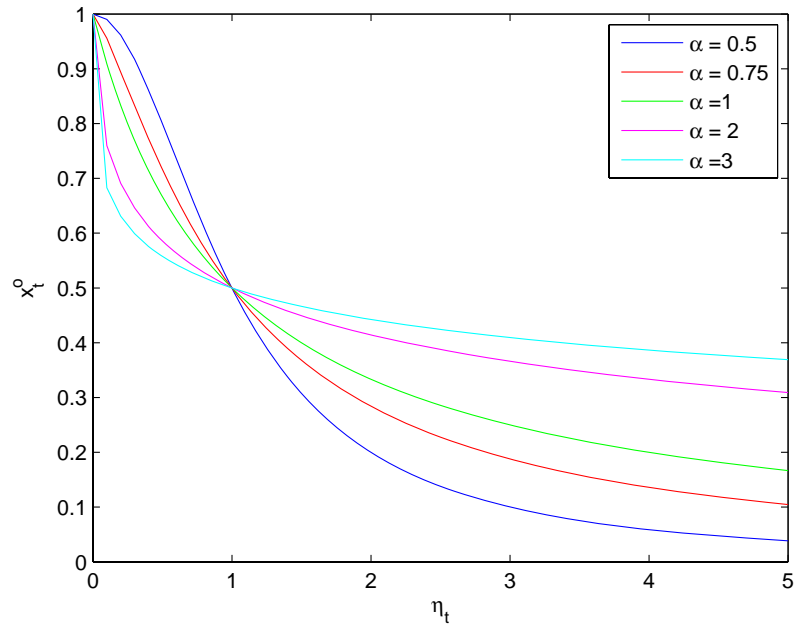


Figure 2: Share of the optimistic agent as a function of η_t .

$$\frac{P_t^m}{D_t} : 2$$

$$\begin{aligned} \varphi_t^1 = & \beta \left(\eta_t^{\frac{1}{\alpha}} (y^t) + 1 \right)^{-\alpha} \times \\ & \times \left[\begin{aligned} & \pi_t^o(h) \left(\left[\frac{\pi_t^p(h)}{\pi_t^o(h)} \right]^{\frac{1}{\alpha}} \eta_t^{\frac{1}{\alpha}} (y^t) + 1 \right)^{\alpha} \exp((1-\alpha) y_h) [1 + \varphi_{t+1}^1] + \\ & \pi_t^o(l) \left(\left[\frac{\pi_t^p(l)}{\pi_t^o(l)} \right]^{\frac{1}{\alpha}} \eta_t^{\frac{1}{\alpha}} (y^t) + 1 \right)^{\alpha} \exp((1-\alpha) y_l) \varphi_{t+1}^1 \end{aligned} \right] \end{aligned}$$

and

$$\begin{aligned} \varphi_t^2 = & \beta \left(\eta_t^{\frac{1}{\alpha}} (y^t) + 1 \right)^{-\alpha} \times \\ & \times \left[\begin{aligned} & \pi_t^o(l) \left(\left[\frac{\pi_t^p(l)}{\pi_t^o(l)} \right]^{\frac{1}{\alpha}} \eta_t^{\frac{1}{\alpha}} (y^t) + 1 \right)^{\alpha} \exp((1-\alpha) y_l) [1 + \varphi_{t+1}^2] + \\ & \pi_t^o(h) \left(\left[\frac{\pi_t^p(h)}{\pi_t^o(h)} \right]^{\frac{1}{\alpha}} \eta_t^{\frac{1}{\alpha}} (y^t) + 1 \right)^{\alpha} \exp((1-\alpha) y_h) \varphi_{t+1}^2 \end{aligned} \right]. \end{aligned}$$

If agents did not distort their beliefs or if they had homogeneous beliefs ($\eta_t = 1, \forall t$), π_t would be the only state variable needed to solve the dynamics of the model. In this case, the equilibrium price-dividend function would be a time constant function of the state beliefs:

$$\varphi_t^m = \varphi^m(\pi_t).$$

Heterogeneity of beliefs, on the contrary, implies that in each period t , a new value of η_t realizes so that the equilibrium price-dividend function is now a function of the stochastic variable η_t :

²Calculus details can be found in the Appendix A 1.

$$\varphi_t^m = \varphi^m(\eta_t, \pi_t; \omega^o, \omega^p)$$

Numerically, this requires solving the functional equations for φ_t^m for the state π_t for each time t , taking η_t as given. Numerical solutions such as Chebyshev collocation methods are quite fast to use and, therefore, suitable also in case of heterogeneity³.

2.1 Long-Run Dynamics

In this section we are interested in understanding the long-run dynamics of the ratio of beliefs, and, therefore, of the consumption share. We impose that heterogeneity among agents persists in the long run to formalize the idea that different opinions and attitudes among the agents are always present and do not characterize only a short-run condition of the economy. In order to model such persistence, we need to impose that both representative agents are *equally wrong* about the realization of the consumption observations. In the long run, with $t \rightarrow \infty$, we can take advantage of the following implications:

1. The full-Bayesian belief of the high-growth state, $\pi_t(h)$, converges to the unconditional probability, $\pi(h) = \frac{1-p_U}{2-p_{hh}-p_U}$;
2. Individual beliefs can be expressed in terms of the stationary probability:

³Matlab code is available upon request.

$$\begin{aligned}\pi^o(h) &= [(1 - \omega^o) \pi(h) + \omega^o], \\ \pi^p(h) &= [(1 - \omega^p) \pi(h)],\end{aligned}$$

so that we can think of $\pi^o(h)$ and $\pi^p(h)$ as the individual unconditional probabilities of the high-growth state;

3. The evolution of the ratio of beliefs takes the following form:

$$\begin{aligned}\eta_t &= \frac{\text{prob}^p(y^t|y^0)}{\text{prob}^o(y^t|y^0)} = \frac{[\text{prob}^p(y_h)]^{\text{prob}(y_h)t} [\text{prob}^p(y_l)]^{\text{prob}(y_l)t}}{[\text{prob}^o(y_h)]^{\text{prob}(y_h)t} [\text{prob}^o(y_l)]^{\text{prob}(y_l)t}} \quad (14) \\ &= \frac{[\pi^p(h)]^{\pi(h)t} [\pi^p(l)]^{\pi(l)t}}{[\pi^o(h)]^{\pi(h)t} [\pi^o(l)]^{\pi(l)t}} = (\eta^*)^t.\end{aligned}$$

Imposing long run survival of both groups implies $\eta^* = 1$ and allows us to compute ω^p as a function of ω^o , so to study the implications of the persistent deviation from the full rationality framework while, technically, keeping limited the number of parameters in the model: for each value of the distortion of the optimistic agent, ω^o , we can derive the corresponding pessimistic distortion such that, in the long run, both types of agents survive. This implies that our model involves only one additional degree of freedom to the standard expected utility framework.

3 Quantitative Analysis

In this section, we estimate a simple two-states mean Markov switching model for the consumption growth process observed in the postwar sample:

$$\log \left(\frac{C_{t+1}}{C_t} \right) = \mu_{\xi_t} + \epsilon_t,$$

with

$$\epsilon_t \sim N(0, \sigma^2),$$

and ξ_t being a 2-states Markov chain whose transition matrix has diagonal terms p_{hh} and p_{ll} .

We then use these values to characterize the dynamics of our simplified two-state discrete version model (1) in a way that is made precise below. Table 1 shows the Maximum-Likelihood estimates for the consumption growth process using quarterly US economy data from the Bureau of Economic Analysis (*BEA*) website⁴. The high-growth state is more persistent and the unconditional probability that the economy finds itself in the high-growth state is higher ($\pi(h) = 0.845$).

⁴A summary of data is available in the Appendix A 2.

μ_h	μ_l	σ	p_{hh}	p_{ll}
0.7244 (0.08)	-0.1999 (0.18)	0.0021 (9.2e-06)	0.9611 (0.0022)	0.8103 (0.0059)

Table 1: Maximum-Likelihood Estimates. Personal Consumption Expenditure (PCE) quarterly data from 1947:1 to 2009:4. Columns 1-3 are expressed in percentage terms. Standard errors are in parentheses.

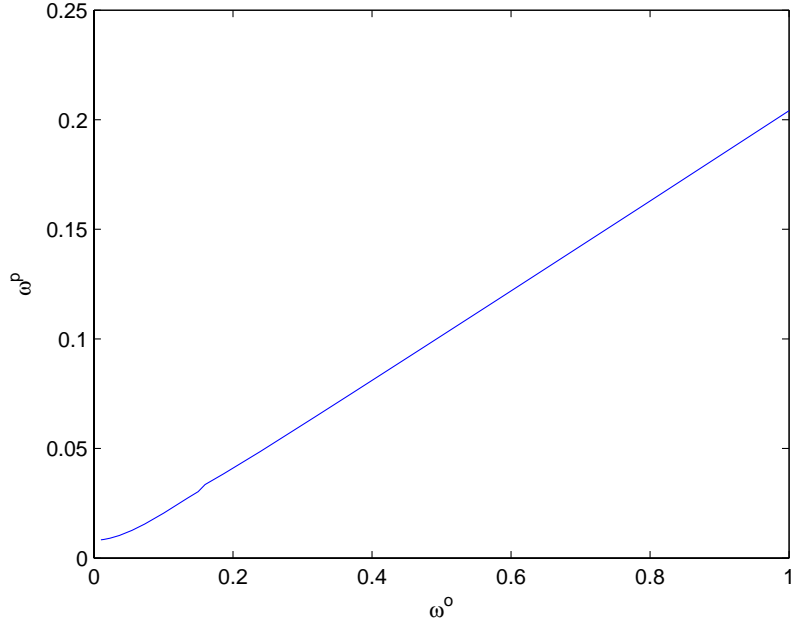


Figure 3: Locus of combinations of ω^o and ω^p such that $\eta^* = 1$ in the long run.

Such asymmetry among the states is reflected in the long-run relation between the distortions of the individual beliefs which makes sure that η_t is equal to 1 in the long run: Figure 3 depicts such relation and shows that for a given value of ω^o , a considerably lower value of the pessimistic distortion is found. In other words, since the economy finds itself in the high-growth state more times, the optimistic believers distort more their beliefs compared to the distortion of the pessimists. If the pessimistic distortion was higher, those beliefs would be driven out of the market because they would be wrong more times than the optimists and the long term ratio of the beliefs, η^* , would become smaller than unity, a possibility that we rule out.

In order to derive the implied price series, we need to discretize the real observations so to make them compatible with our simple discrete model. A natural choice is to consider the observation y_t as y_h (y_l) if $y_t < y_\mu$ ($y_t > y_\mu$), where y_μ is the value corresponding to the inverse cumulative function such that

$$prob(y_t < y_\mu) = \pi(l), \forall t.$$

In each time t , given η_t and π_t , the model can generate the implied consumption shares and equilibrium prices. Figure 4 presents a comparative graph where the dynamics of the optimistic share is plotted for different values of the distortion ω^o .

Higher levels of distortion imply more significant switching among consumption shares. In the extreme case of no distortion, consumption shares are constant and equal to 0.5. Figure 5 shows a similar comparative exercise, where equilibrium shares and prices are plotted for different values of distortions (vertically) and risk aversion (inside each panel).

As already noted above, risk aversion smooths the dynamics of the switching of the consumption share for different levels of risk aversion while higher levels of distortions have the opposite effect. Price-dividend ratios appear to be very sensitive to the switching of shares if risk aversion is low. High biases in beliefs and low risk aversion are able to imply very volatile prices, which appear very low (even implausible negative for some combinations of the parameters) in periods in which pessimists get the biggest consumption share and very high in growth periods.

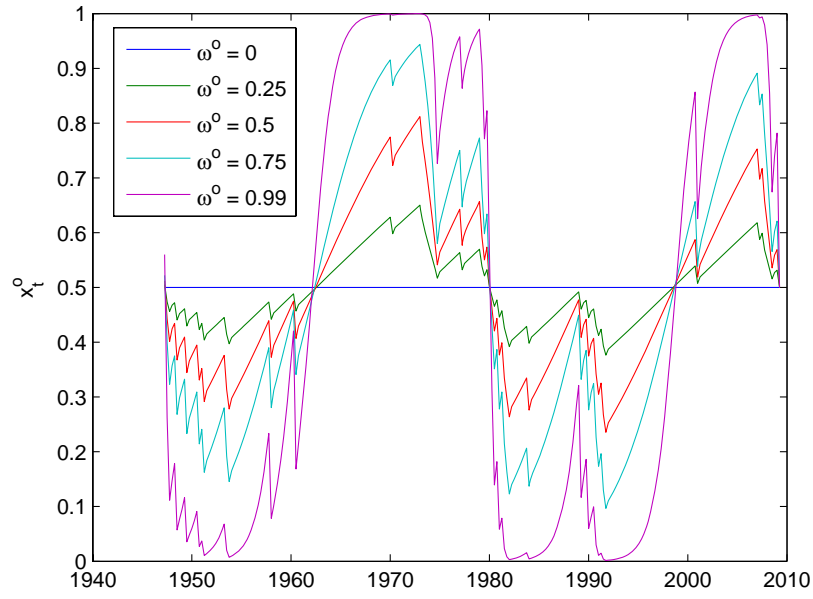


Figure 4: Time series of share of the optimistic agent for different intensities of distortion. ($\alpha = 4.5$, $\beta = 0.995$).

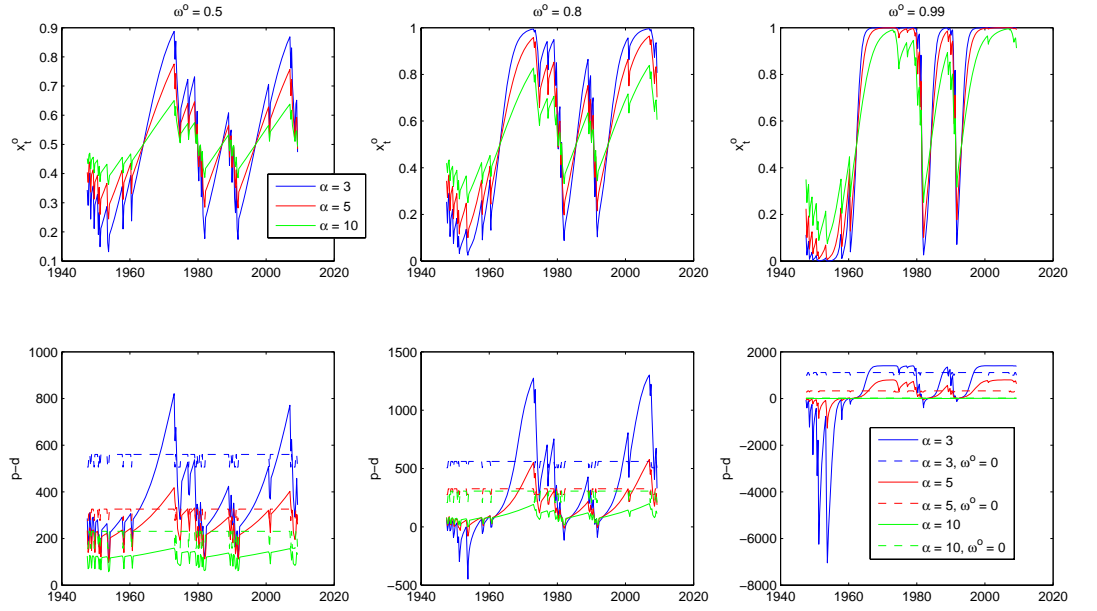


Figure 5: Comparative analysis of consumption shares and price-dividend ratios for different risk aversion and intensity of distortion parameters. ($\beta = 0.995$).

The implications of the model in terms of prices for different values of risk aversion and distortions can be directly compared with real data. Low levels of risk aversion can produce very volatile prices in presence of heterogeneity. This suggests that heterogeneity plays an important role towards the explanation of the equity premium puzzle because even with small risk aversion the model is able to generate price fluctuations close to the observed ones. Figures 6 and 7 show such comparisons and present the correlation coefficient between data and the implied series for several values of risk aversion and make clear that the presence of heterogeneity of beliefs amplifies the effect of low values of risk aversion, helping in matching the medium term fluctuations. Such effect would not be captured in a similar simple model with homogeneous beliefs.

We also observe that there is substitutability between risk aversion and distortion: higher risk aversion produces lower but less volatile price-dividend ratios, while higher distortion in beliefs causes prices to be lower and more volatile. This fact suggests that high values of distortion in beliefs can substitute risk aversion so that a good matching of the series can be obtained with low values of risk aversion. Heterogeneity in this sense can help in resolving the equity premium puzzle: low price-dividend ratios usually imply expected higher returns.

As a baseline case, we choose a calibration which appears quite satisfactory in matching the level and the medium-long term fluctuations in prices, where we set α equal to 8 and ω^o equal to 0.8. Figure 8 shows the comparison between the data (green dashed line) and the implied prices (blue solid line). We also show the implied series with the same level of risk

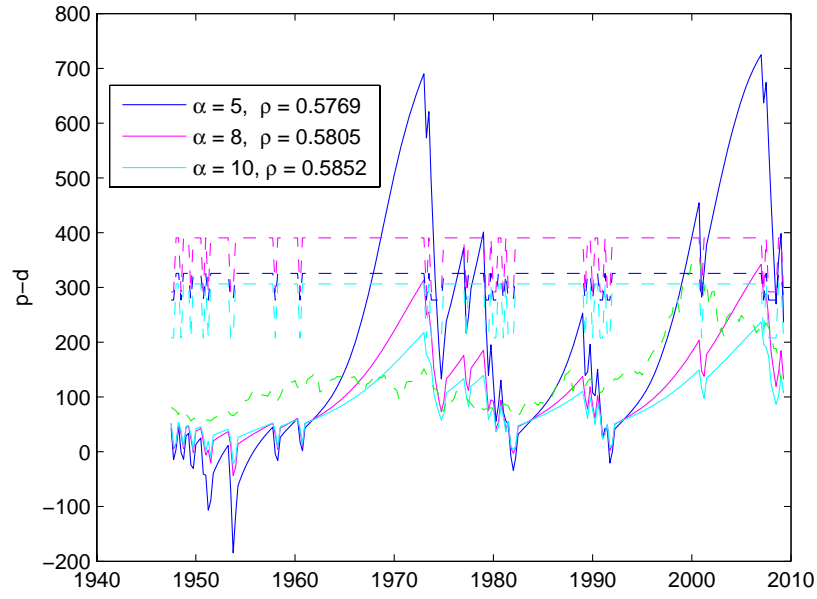


Figure 6: Implied price-dividend ratio for different values of risk aversion. The dashed green line represents the observed series. ($\omega^o = 0.9, \beta = 0.995$).

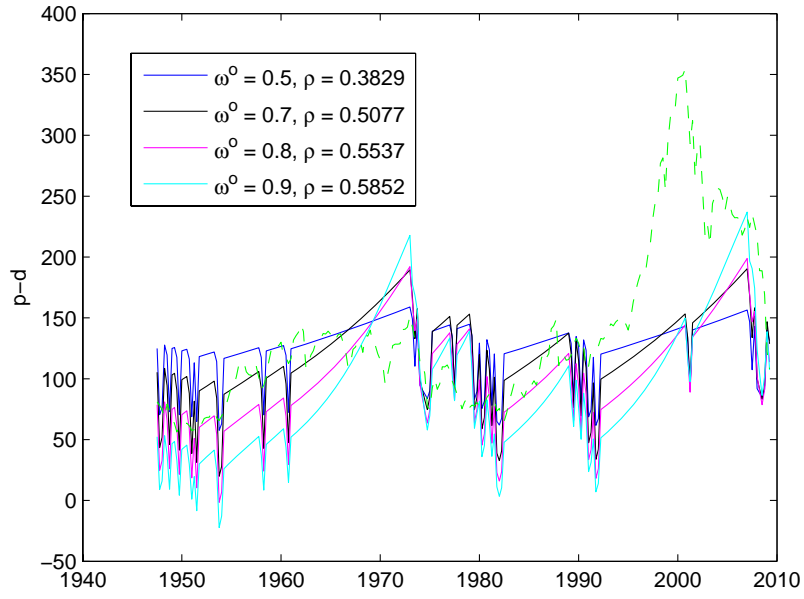


Figure 7: Implied price-dividend ratio for different values of distortions. The green dashed line represents the observed series. ($\alpha = 10, \beta = 0.995$).

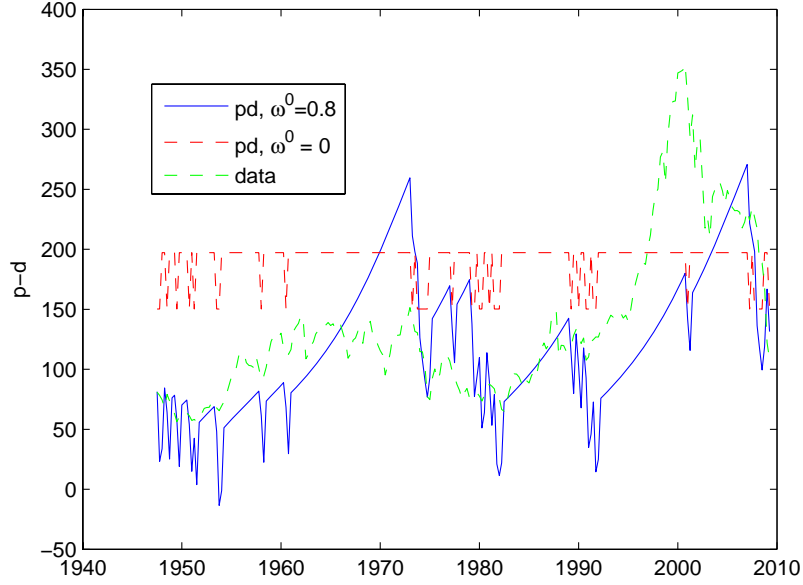


Figure 8: Implied and observed price-dividend ratios. The red dashed line represents the implied series with no distortion in beliefs. The green dashed line represents the observed series. ($\alpha = 8$, $\omega^o = 0.8$, $\gamma = 1$, $\beta = 0.995$).

aversion without heterogeneity in beliefs (red dashed line). The correlation coefficient, ρ , is 0.5580, that can be considered quite satisfactory given the simplistic structure of the model.

The corresponding dynamics of the share of the optimists is represented in Figure 9. Clearly, the dynamics of the shares among agents determines the waves of prices: when the optimistic group is the majority, their consumption shares cause prices to increase and the reverse happens when the pessimistic share is the greatest. At the beginning of the year 2010, the share of the optimistic is still greater than the pessimistic one, but it is rapidly decreasing. We know that during the current prolonged financial distress,

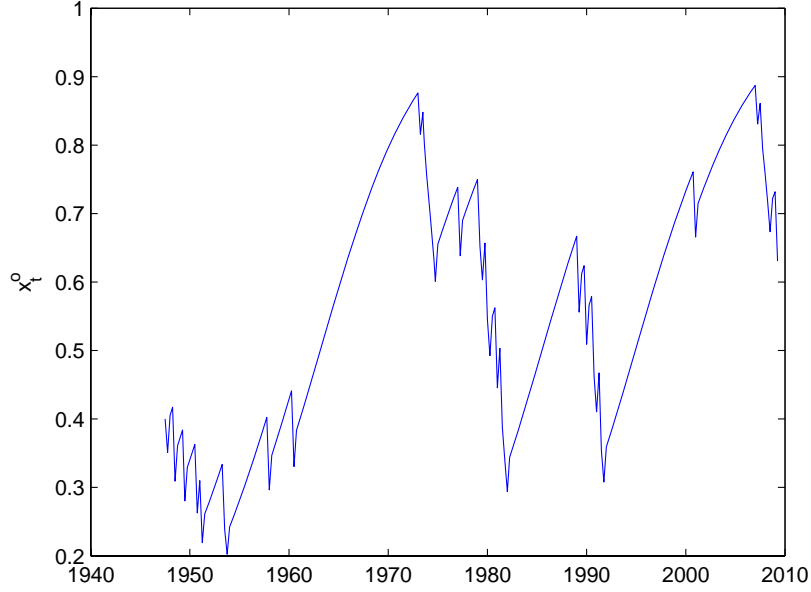


Figure 9: Share of the optimistic agents. ($\alpha = 8, \omega^o = 0.8, \gamma = 1, \beta = 0.995$)).

in the first half of 2011, there has been a lot of uncertainty regarding the consumers' sentiment and the possibility of a new fast recovery. The figures suggests, however, that fast recovery in the economy and in consumers' sentiment has been experienced after years of extreme pessimism, as if it is likely to observe still unstable periods of recession before experiencing a new stable optimism view of the economy.

We conclude that this simple model is able to imply persistent effects of changing confidence levels and can be thought as a simple formalized counterpart of the idea of confidence multiplier of Akerlof and Shiller (2009): *"Changes in confidence will result in changes in income and confidence in the next round, and each of these changes will in turn affect income and*

confidence in yet further rounds” (p. 16).

4 Testable moments restrictions

In this section we focus on the testability of the equilibrium conditions of the model simple moment conditions that can be derived from the model with heterogeneous expectations. Let M be the set of assets. From (7), the time t price of an asset $m \in M$ must satisfy the following Euler for agent i :

$$1 = \beta \mathbb{E}_t^i \left[\left(\frac{C_t^i}{C_{t+1}^i} \right)^{-\alpha} R_{t+1}^m \right], m \in M, i \in \{o, p\} \text{ and } t \in T, \quad (15)$$

where $R_{t+1}^m = \frac{P_{t+1}^m + D_{t+1}^m}{P_t^m}$ is the gross return of asset m .

Substituting the share of optimistic agent, eq. (15) becomes:

$$\begin{aligned} 1 &= \beta \mathbb{E}_t^o \left[\left(\frac{(\eta_t)^{\frac{1}{\alpha}} + 1}{(\eta_{t+1})^{\frac{1}{\alpha}} + 1} \right)^{-\alpha} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} R_{t+1}^m \right] \\ &= \beta \mathbb{E}_t^o f(\eta_{t+1}, y_{t+1}, R_{t+1}^m), \end{aligned}$$

where $y_{t+1} = \log \frac{C_{t+1}}{C_t}$. It follows that

$$\begin{aligned} 1 &= \beta \mathbb{E}_t^o f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \\ &= \beta \sum_{y_{t+1}} f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \text{prob}^o(y_{t+1}|y^t). \end{aligned} \quad (16)$$

It is easy to show that

$$\text{prob}^o(y_{t+1}|y^t) = (1 - \omega^o) \text{prob}(y_{t+1}|y^t) + \omega^o \text{prob}(y_{t+1}|S_{t+1} = h), \quad (17)$$

where $\text{prob}(y_{t+1}|y^t)$ is the full-Bayesian probability⁵. Substituting this definition in (16) gives the condition:

$$1 = \sum_{y_{t+1}} f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \{ (1 - \omega^o) \text{prob}(y_{t+1}|y^t) + \omega^o \text{prob}(y_{t+1}|S_{t+1} = h) \}, \quad (18)$$

in which we need to work out more explicitly the term $\text{prob}(y_{t+1}|S_{t+1} = h)$.

By the properties of the conditional probabilities the following equalities can be derived:

$$\begin{aligned} \text{prob}(y_{t+1}|S_{t+1} = h) &= \frac{\text{prob}(y_{t+1}, S_{t+1} = h)}{\text{prob}(S_{t+1} = h)} \\ &= \frac{\text{prob}(S_{t+1} = h|y_{t+1}) \text{prob}(y_{t+1})}{\text{prob}(S_{t+1} = h)} \\ &= \frac{\pi_{t+1,t+1}(h) \text{prob}(y_{t+1})}{\pi(h)}, \end{aligned} \quad (19)$$

so that substituting the term back into (18) we have:

$$\begin{aligned} 1 &= \sum_{y_{t+1}} f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \left\{ (1 - \omega^o) \text{prob}(y_{t+1}|y^t) + \omega^o \frac{\pi_{t+1,t+1}(h) \text{prob}(y_{t+1})}{\pi(h)} \right\} \\ &= (1 - \omega^o) \sum_{y_{t+1}} \text{prob}(y_{t+1}|y^t) f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \\ &\quad + \omega^o \sum_{y_{t+1}} \text{prob}(y_{t+1}) f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \frac{\pi_{t+1,t+1}(h)}{\pi(h)} \\ &= (1 - \omega^o) E_t f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) + \frac{\omega^o}{\pi(h)} \mathbb{E} f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \pi_{t+1,t+1}(h). \end{aligned}$$

⁵See Appendix A 3 for details.

Let z_t be a q -dimensional vector of instrumental variables that are in the agents' information set. The equilibrium condition in terms of unconditional expectations can be defined as follows:

$$\begin{aligned} z_t = & (1 - \omega^o) \mathbb{E}_t \{ f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \otimes z_t \} + \\ & + \frac{\omega^o}{\pi(h)} \mathbb{E} \{ f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \pi_{t+1,t+1}(h) \} \otimes z_t, \end{aligned}$$

where \otimes denotes the Kronecker product and $\pi_{t+1,t+1}(h)$ is the full-Bayesian posterior probability of the high-mean state, at time $t + 1$, after observing y_{t+1} , that is, $\pi_{t+1,t+1}(h) = \text{prob}[S_{t+1} = h \mid y^{t+1}]$. Taking the unconditional expectation of both sides, we have:

$$\begin{aligned} 0 &= \mathbb{E} \left\{ \begin{aligned} & (1 - \omega^o) \mathbb{E}_t \{ f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \otimes z_t \} + \\ & + \frac{\omega^o}{\pi(h)} \mathbb{E} \{ f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \pi_{t+1,t+1}(h) \} \otimes z_t - z_t \end{aligned} \right\} \\ &= (1 - \omega^o) \mathbb{E} \{ f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \otimes z_t \} + \\ & \quad + \frac{\omega^o}{\pi(h)} \mathbb{E} \{ f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \pi_{t+1,t+1}(h) \} \otimes \mathbb{E} z_t - \mathbb{E} z_t \end{aligned}$$

and the sample equivalent of the equilibrium condition can be defined as:

$$0 = \frac{1}{T} \sum_{t=1}^T \left\{ \begin{aligned} & (1 - \omega^o) \{ f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \otimes z_t \} + \\ & + \frac{\omega^o}{\pi(h)} \{ f(\eta_{t+1}, y_{t+1}, R_{t+1}^m) \pi_{t+1,t+1}(h) \} \otimes \frac{1}{T} \sum_{t=1}^T z_t - z_t \end{aligned} \right\}.$$

In our model the ratio of beliefs, η_{t+1} , is a constant function of y_{t+1} .

Indeed,

$$\eta_{t+1} = \frac{\prod_{j=1}^{t+1} prob^p(y_j|y_{j-1})}{\prod_{j=1}^{t+1} prob^o(y_j|y_{j-1})}$$

so that, at each time t , η_t is known and a function of y_t given the distortion parameters.

Therefore, the Generalized Method of Moments testable condition can be written as:

$$0 = \frac{1}{T} \sum_{t=1}^T \left\{ \begin{array}{l} (1 - \omega^o) f(y_{t+1}, R_{t+1}^m; \omega^o) \otimes z_t + \\ \frac{\omega^o}{\pi(h)} f(y_{t+1}, R_{t+1}^m; \omega^o) \pi_{t+1,t+1}(h) \otimes \frac{1}{T} \sum_{t=1}^T z_t - z_t \end{array} \right\}.$$

In order to estimate the parameters α , ω^o and β , we use the nondurable plus services series of personal consumption expenditure, available at the Bureau of Economic Analysis (*BEA*) website. Nominal market returns and risk free rates are available in the CRSP website. We used quarterly data from 1947:2 to 2009:2. Nominal values are converted into real quantities by dividing by the implicit price deflator associated with the consumption series⁶.

We consider different combinations of instrumental variables, made of lagged values of consumption and lagged returns. A first round of consistent but inefficient estimates is obtained using the identity matrix as the weighting matrix. The consistent estimates are used to construct the efficient weighting matrix. Table 2 reports the estimates, the column DF

⁶A detailed description of the data is in the Appendix A 2.

denotes the degrees of freedom or the number of overidentifying restrictions, and *prob* represents the probability that a $\chi^2(DF)$ random variate is less than the computed value of the test statistic under the hypothesis that the model equilibrium conditions are satisfied.

<i>INST</i>	α	ω^o	β	<i>DF</i>	<i>Prob</i>
<i>const; y₋₁; y₋₂; R₋₁</i>	3.0991 (5.0869)	0.1000 (0.0505)	0.9925 (0.0329)	1	0.9268
<i>const; y₋₁; y₋₂; R₋₁ * y₋₁; R₋₁ * y₋₂</i>	3.1055 (13.1846)	0.1000 (0.2269)	0.9900 (0.0761)	2	0.1729
<i>const; y₋₁²; R₋₁²; R₋₁² * y₋₁²</i>	3.0986 (17.1676)	0.1000 (0.2652)	0.9902 (0.1069)	1	0.1981
<i>const; y₋₁²; R₋₁²; y₋₁² * R₋₁²; y₋₂²</i>	3.0995 (17.0850)	0.1000 (0.2651)	0.9901 (0.1064)	2	0.1497

Table 2: Generalized Methods of Moments estimates with overidentified conditions. Standard errors are in parentheses.

$INST$	α	ω^o	β
$const; y_{-1}; R_{-1}$	3.0995 (5.1169)	0.1000 (0.0581)	0.9923 (0.0330)
$const; y_{-1}; y_{-2}$	3.1070 (13.3396)	0.1000 (0.2599)	0.9898 (0.0766)
$const; y_{-1}^2; y_{-2}^2$	3.0797 (61.9299)	0.1000 (2.7636)	0.9901 (0.5304)
$const; y_{-1}^2; R_{-1}^2$	3.0885 (17.4155)	0.1640 (0.4155)	0.9862 (0.1208)

Table 3: Generalized Methods of Moments estimates in the case of exactly identification. Standard errors are in parentheses.

The estimates of α , ω^o and β appear similar in the different cases. The J -test suggests that the model is correctly specified. However, the standard errors of the risk aversion parameter are usually very high. We should recall, indeed, that we are considering a standard utility specification, where the risk aversion parameter is constrained to be the inverse of the elasticity of substitution. Such tension has been emphasized in the literature and, starting from Epstein and Zin (1989), more flexible frameworks have been provided. However, we do not address this issue here: we want to concentrate exclusively on the role of distortion of beliefs and we keep the model as simple as possible in order to make the interpretation easier. The estimates of the distortion parameter are usually positive, but not extremely high. Requiring the discount factor to be less than unity, is, in most cases, a binding constraint. This feature is common among expected utility models suggesting that this framework have some problems in fitting the levels of asset returns. Similar results are obtained in case of exact identification, which is summarized in Table 3.

5 Conclusions

We propose a simple asset pricing model with (risk averse) heterogeneous agents. Heterogeneity is modelled in a very simple way, captured by one single parameter. In this way, we want to propose a practical alternative to complex heterogeneous agents models, which, starting from Brock and

Hommes (1997), have been able to imply interesting and complex price dynamics at the expenses of the robustness of the validation: such models, indeed, often rely on numerous parameters which are usually calibrated. Furthermore, we are not only interested in the ability of the model to produce realistic dynamics (like waves of optimism and pessimism, volatility clustering, etc.), but we would also like to build a model which allows to reproduce the observed price-dividend ratios during the postwar sample.

Agents have access to the same information, but they interpret it with different attitudes: the optimistic group believes that good times are more likely than full-Bayesian learners would, while the pessimists increase the probability of bad times happening. In order to control the effect of such types of heterogeneity, we consider a simple complete market: in our economy, there are two possible states and two Arrow-Debreu assets which pay the aggregate dividend if a specific predefined state of nature realizes. A key assumption of our model is that heterogeneity is not going to be resolved: pessimists and optimists are equally wrong about the probability of future states of nature, they both survive in the long run so that prices reflect the temporary deviation from the long-run persistence of heterogeneity.

The model is able to reproduce the waves of optimism and pessimism observed in the dynamics of asset prices. The amplitude of such waves is influenced by the risk aversion parameter and the distortion in beliefs formation imposed on the agents. Low levels of risk aversion determine a noticeable amplification effect which is peculiar to the case of an economy with heterogeneous agents: risk neutral investors tend to make more extreme investment decisions, which, if based on wrong beliefs, lead them to be

driven out of the market. Furthermore, the more agents are (equally) wrong about the probabilities of future realizations, the more price fluctuations are volatile. This suggests that heterogeneity in the form of persistent different attitudes is relevant in understanding the main forces driving asset prices and can constitute a useful alternative to the existing heterogeneity models in terms of robustness and validation.

We also present an attempt to estimate the model with GMM methods and we show that it is possible to derive testable moment conditions.

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A Appendix

A.1 The price-dividend ratio

We consider the price function (7) in terms of the optimistic agent and we substitute on it the utility function form and eq. (12) for C_t^o :

$$\begin{aligned}
P_t^m(y^t) &= \beta \sum_{y^{t+1}} \text{prob}^o(y_{t+1}|y_t) \frac{u'(C_{t+1}^o(y^{t+1}))}{u'(C_t^o(y^t))} (P_{t+1}^m(y^{t+1}) + D_{t+1}^m(y^{t+1})), \\
&= \beta \sum_{y^{t+1}} \text{prob}^o(y_{t+1}|y_t) \frac{(C_{t+1}^o(y^{t+1}))^{-\alpha}}{(C_t^o(y^t))^{-\alpha}} (P_{t+1}^m(y^{t+1}) + D_{t+1}^m(y^{t+1})), \\
&= \beta \sum_{y^{t+1}} \text{prob}^o(y_{t+1}|y_t) \frac{\left(\frac{1}{(\eta_{t+1}(y^{t+1}))^{\frac{1}{\alpha}+1}} D_{t+1}(y^{t+1})\right)^{-\alpha}}{\left(\frac{1}{(\eta_t(y^t))^{\frac{1}{\alpha}+1}} D_t(y^t)\right)^{-\alpha}} (P_{t+1}^m(y^{t+1}) + D_{t+1}^m(y^{t+1})), \\
&= \beta \sum_{y^{t+1}} \text{prob}^o(y_{t+1}|y_t) \left(\frac{\eta_t^{\frac{1}{\alpha}}(y^t) + 1}{\eta_{t+1}^{\frac{1}{\alpha}}(y^{t+1}) + 1}\right)^{-\alpha} \times \\
&\quad \times \left(\frac{D_{t+1}(y^{t+1})}{D_t(y^t)}\right)^{-\alpha} (P_{t+1}^m(y^{t+1}) + D_{t+1}^m(y^{t+1})),
\end{aligned} \tag{20}$$

where $D_{t+1}^m(j) = D_{t+1}$ if $m = 1(2), \wedge j = h(l)$, zero otherwise.

Dividing both sides by D_t and denoting with $\varphi_t^k = P_t^k/D_t$ the price-dividend ratio we get

$$\begin{aligned}
\varphi_t^m &= \beta \left(\eta_t^{\frac{1}{\alpha}}(y^t) + 1\right)^{-\alpha} \times \\
&\quad \times \left[\begin{aligned} &\pi_t^o(j) \left(\left[\frac{\pi_t^p(j)}{\pi_t^o(j)}\right]^{\frac{1}{\alpha}} \eta_t^{\frac{1}{\alpha}}(y^t) + 1\right)^{\alpha} \exp((1-\alpha)y_j) [1 + \varphi_{t+1}^m] + \\ &\pi_t^o(k) \left(\left[\frac{\pi_t^p(k)}{\pi_t^o(k)}\right]^{\frac{1}{\alpha}} \eta_t^{\frac{1}{\alpha}}(y^t) + 1\right)^{\alpha} \exp((1-\alpha)y_k) \varphi_{t+1}^m \end{aligned} \right]
\end{aligned} \tag{21}$$

for $j \neq k, \forall m$.

A.2 Data description

We have used quarterly real observations. Consumption is derived using the quarterly series of Personal Consumption Expenditure from the Bureau of Economic Analysis (*BEA*) website. Value weighted portfolio returns are from the *CRSP* dataset. Nominal values are deflated using the Implicit Price Deflator relative to the consumption series from the *BEA* website. Data span the period 1947:2-2010:2.

A.3 Appendix

We present here the algebraic steps which prove the equivalence stated in eq. (17):

$$\begin{aligned}
prob^o(y_{t+1}|y^t) &= prob(y_{t+1}|S_{t+1} = h) \pi_t^o(h) + prob(y_{t+1}|S_{t+1} = l) \pi_t^o(l) \\
&= prob(y_{t+1}|S_{t+1} = h) [(1 - \omega^o) \pi_t(h) + \omega^o] + \\
&\quad prob(y_{t+1}|S_{t+1} = l) [(1 - \omega^o) \pi_t(l)] \\
&= prob(y_{t+1}|S_{t+1} = h) (1 - \omega^o) \pi_t(h) + \\
&\quad prob(y_{t+1}|S_{t+1} = l) [(1 - \omega^o) \pi_t(l)] + \\
&\quad \omega^o prob(y_{t+1}|S_{t+1} = h) \\
&= (1 - \omega^o) [prob(y_{t+1}|S_{t+1} = h) \pi_t(h) + prob(y_{t+1}|S_{t+1} = l) \pi_t(l)] + \\
&\quad \omega^o prob(y_{t+1}|S_{t+1} = h) \\
&= (1 - \omega^o) prob(y_{t+1}|y^t) + \omega^o prob(y_{t+1}|S_{t+1} = h).
\end{aligned}$$

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Abstract

We consider a simple consumption-based asset pricing model with two types of investors who have access to the same observations but who use different updating rules to infer information about the growth state of the economy.

In particular, we consider an optimistic and pessimistic group of agents who use distorted Bayesian updating rules. The aim of the work is to understand to what extent the interaction of such distorted Bayesian rules can explain low and medium frequency characterization of dynamics movements observed in the price dividend ratios and can give rise endogenously to waves of pessimism and optimism which are associated with sustained asset price booms and busts. The analysis shows that heterogeneity in ambiguity loving/aversion preferences appears to be an important factor to capture medium-frequency waves observed on asset prices.

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